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Published in:
Physics Letters B

DOI:
10.1016/0370-2693(86)90207-8

THE SINGLE-PARTICLE INTERACTION IN NUCLEAR MATTER VIA THE RELATIVISTIC DIRAC–BRUECKNER APPROACH

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Received 2 January 1986; revised manuscript received 3 March 1986

Within the relativistic Dirac–Brueckner approach and starting from a one-boson-exchange interaction, the nucleon selfenergy is calculated above the nuclear-matter Fermi sea. The effects of Pauli blocking and energy dispersion are studied. At low energy we see a dominance of the Pauli blocking whereas at nucleon energies up to 250 MeV the dispersive effect still has a very large influence on the single-particle interaction. From the selfenergy a Schrödinger optical potential is deduced, for which the DB results nicely agree with empirical values. The density dependence of this optical potential compares well with earlier calculations.

Recently relativistic Dirac calculations have been very successful in describing elastic proton–nucleus scattering in the energy range of 300–800 MeV [1–3]. In this approach the Dirac equation is taken as the relevant wave equation and the potentials are specified in terms of their Lorentz character. Generally, large Lorentz scalar and vector potentials are found of hundreds of MeV each but with opposite sign. Most of the calculations so far are based on the impulse approximation, where the free NN interaction is taken as input for the determination of the potentials. If the empirical NN phase shifts are used directly, only the onshell structure of the potentials can be deduced, which leads to several ambiguities. Therefore a (meson) theoretical description of the free NN interaction is preferred. Tjon and Wallace [4] have shown in an elaborate calculation based on meson theory, including isobar degrees of freedom, that even below 200 MeV proton scattering can be described reasonably well, although they somewhat overpredict the cross sections.

This success of the relativistic impulse approximation contradicts in fact with the conclusions of microscopic calculations in non-relativistic theory. In particular Von Geramb and coworkers have demonstrated [5] that medium effects have an important influence on the optical potential even at higher energies. It is therefore interesting to study medium effects, i.e. Pauli blocking and dispersive effects on the single-particle energies, in the framework of relativistic Dirac theory. The effect of Pauli blocking has recently been studied by Horowitz [6], who concluded that in relativistic calculations the effect is smaller than in non-relativistic calculations. In this letter we want to investigate the both aforementioned medium effects in nuclear matter and present a Dirac–Brueckner (DB) calculation for incoming particles above the Fermi sea, based on a one-boson-exchange (OBE) interaction. Similar calculations have been presented by Shakin and coworkers [7], though their Brueckner calculattions are not fully selfconsistent. We followed the method of solution of Horowitz and Serot [8], which differs considerably from the work of ref. [7].

Our DB calculations start with the selfconsistent solution of a Thompson equation [9] for two nucleons in a nuclear medium. This equation is a three-dimensional reduction of the Bethe–Salpeter equation and is very similar to the more familiar Blankenbecler–Sugar approach, the difference being the precise form of the two-nucleon Green function (for which an infinite set of covariant and unitary formulations is possible). The Thompson equation can be written as

$$\langle \boldsymbol{p}'' s''_{12} | \Gamma | \boldsymbol{p}\, s_{12} \rangle = \langle \boldsymbol{p}'' s''_{12} | U | \boldsymbol{p}\, s_{12} \rangle + \sum_{s'_{12}} \int \mathrm{d}\boldsymbol{p}' \langle \boldsymbol{p}'' s''_{12} | U | \boldsymbol{p}' s'_{12} \rangle \times \frac{Q(\boldsymbol{p}', \boldsymbol{P}, s^*)}{E^{*2}_{p'} (\frac{1}{2}\sqrt{s^*} - E^*_{p'} + \mathrm{i}\epsilon)} \langle \boldsymbol{p}' s'_{12} | \Gamma | \boldsymbol{p}\, s_{12} \rangle , \qquad (1)$$

where U gives the OBE interaction and Q is a relativistic Pauli exclusion operator which depends not only on the relative momentum in the two-particle center of momentum frame $\boldsymbol{p}'$, but also on the total momentum $\boldsymbol{P}$ in the nuclear-matter rest frame and the total invariant mass s^*; s_{12} stands for the spin values of particle 1 and 2, projected along the z-axis. The "stars" (*) in the equation represent the influence of the nucleon selfenergy Σ, which itself depends on the effective t-matrix Γ via

$$\Sigma(k) = -\mathrm{i} \int [\mathrm{tr}(G\Gamma) - G\Gamma] , \qquad (2)$$

where Γ now has been transformed to the nuclear-matter rest frame. $\Sigma(k)$ can be expanded in its general form:

$$\Sigma(k) = \Sigma_\mathrm{s}(k) - \gamma_0 \Sigma_0(k) + \boldsymbol{\gamma} \cdot \boldsymbol{k} \Sigma_\mathrm{v}(k) , \qquad (3)$$

which enables us to define

$$k^*_\mu = k_\mu + \delta_{\mu 0} \Sigma_0 , \quad E^*_\mathrm{k} = (\boldsymbol{k}^2 + m^{*2})^{1/2} ,$$
$$m^* = m + \Sigma_\mathrm{s} - m^* \Sigma_\mathrm{v} , \qquad (4)$$

where the weakly momentum-dependent $\Sigma(k)$ is approximated by its value on the Fermi-surface. The replacement of the selfenergy contribution by constants simplifies the solution of eq. (1) considerably. In this model (of which more details can be found in ref. [8]) the interaction U contains effective Dirac spinors:

$$u(p^*, \sigma) = \left(\frac{E^* + m^*}{2m^*} \right)^{1/2} \begin{pmatrix} 1 \\ \dfrac{\boldsymbol{\sigma} \cdot \boldsymbol{p}}{E^* + m^*} \end{pmatrix} \chi_\sigma , \qquad (5)$$

which results in a "dressed" and density-dependent interaction. To solve eq. (1), we used the Padé approximant method. Instead of expanding eq. (1) into a partial wave-helicity frame, we calculated the equation in full momentum–spin space. Therefore the three-dimensional integral equation has been reduced to a two-dimensional one by means of the rotational symmetry relation

$$\langle p' \theta \psi s'_1 s'_2 | \Gamma | p\, 0\, 0\, s_1 s_2 \rangle = \exp[\mathrm{i}\psi(s_1 + s_2 - s'_1 - s'_2)] \times \langle p' \theta\, 0\, s'_1 s'_2 | \Gamma | p\, 0\, 0\, s_1 s_2 \rangle . \qquad (6)$$

The aformentioned transformation of the effective t-matrix from the two-particle CM frame to the medium rest frame is achieved by projecting Γ on five Lorentz-invariant interaction matrices:

$$\Gamma = \sum_\alpha \Gamma^\alpha f^\alpha_{(1)} \cdot f^\alpha_{(2)} , \qquad (7)$$

with

$$f^\alpha_{(i)} = \{ 1, \gamma^\mu_{(i)}, \sigma^{\mu\nu}_{(i)}, \gamma_5 \gamma^\mu_{(i)}, \gamma_5 \not{q} \} . \qquad (8)$$

We use a pseudo-vector interaction instead of pseudoscalar, in agreement with our choice for the one-pion-exchange coupling. The ambiguity that appears here has been discussed in refs. [4,10], which also favour the pseudo-vector coupling.

Our OBE interaction contains, $\pi, \omega, \rho, \epsilon, \eta$ and δ-exchange, for which the lagrangians of ref. [11] were used. The parameters of the interaction are given in table 1. A monopole form factor $\Lambda^2/(\Lambda^2 + q^2)$ is added to the vertices. Since no isobar degrees of freedom are included, we are restricted to nucleon energies below 300 MeV. In the next future we will present calculations including isobars, which enable us to investigate a wider energy range. Solving the Thompson equation, the OBE interaction gives a very good description of the free NN phase shifts, cross sections and polarisation data. With respect to the sa-

Table 1
Parameters of the OBE interaction

Meson	Mass(MeV)	I, J^P	$g^2_\alpha/4\pi$	f_α/g_α
π	139	$1,0^-$	14.16	
ω	784	$0,1^-$	11.7	0.0
ρ	764	$1,1^-$	0.43	5.1
ϵ	571	$0,0^+$	7.8	
η	550	$0,0^-$	2.0	
δ	962	$1,0^+$	1.43	
Λ	1.3 a)			

a) $\mathrm{Mass}^2(\mathrm{GeV}^2)$.

turation properties of nuclear matter, the DB approach turns out to be very successful, which has already been pointed out by Shakin et al. [7] and by Machleidt and Brockmann [12]. As we presented elsewhere [13] our calculation gives a binding energy of $E_B = -14$ MeV at a saturation density of $\rho_0 = 0.16\ \mathrm{fm}^{-3}$. This is closer to the empirical values than conventional non-relativistic Brueckner calculations.

Within the DB model that we describe here, single-particle selfenergies above and below the Fermi surface can be calculated in exactly the same way. The dressed nucleon propagators that enter in the calculation are constructed by using the constants Σ_s, Σ_0 and Σ_v, which are obtained at the Fermi surface. In fact, since the zero component of the four-momentum plays no role in the Thompson equation, only

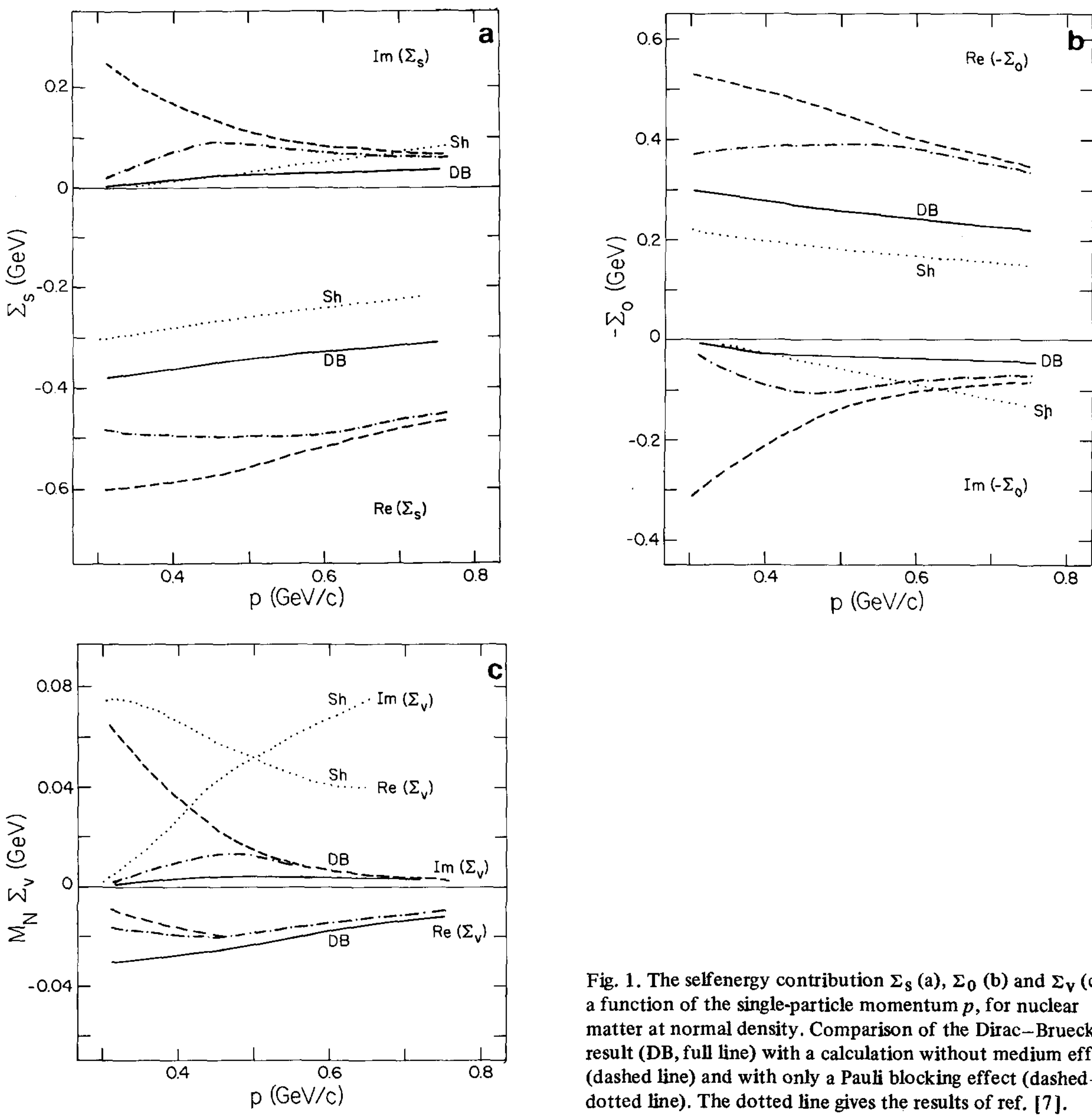

Fig. 1. The selfenergy contribution Σ_S (a), Σ_0 (b) and Σ_V (c) as a function of the single-particle momentum p, for nuclear matter at normal density. Comparison of the Dirac–Brueckner result (DB, full line) with a calculation without medium effects (dashed line) and with only a Pauli blocking effect (dashed–dotted line). The dotted line gives the results of ref. [7].

m^* acts as iteration- or selfconsistency parameter, being only dependent on the density of the medium, but not on the velocity of the single particle. This leads to the procedure in which eqs. (1) and (2) are solved iteratively for particle 1 at the Fermi surface (and particle 2 integrated over the Fermi sea) until a selfconsistent value for m^* is obtained. This m^* serves then as input for another solution of eqs. (1) and (2), the momentum of particle 1 being k_1, leading to the selfenergy $\Sigma(k_1)$. (Note that the three-momentum k of a particle is not effected by medium corrections in our model, see eq. (4).) The major difference of the selfenergy calculation for particles below or above the Fermi surface, is that below the Fermi level $\Sigma(k)$ is a real, above the sea it becomes a complex quantity.

Our results for $\Sigma(k)$ are presented in figs. 1a–1c, where at saturation density $\Sigma_s(k)$, $-\Sigma_0(k)$ and $m_N\Sigma_v(k)$ are displayed separately. The full DB calculation is compared with a calculation where $m^* = m_N$ is assumed (dashed–dotted line), so in which only the Pauli blocking is taken into account, and a calculation with a free t-matrix, without any medium effect (dashed line). It is clearly seen that the Pauli blocking is very important just above the Fermi surface, while at $k = 0.7$ GeV/c only a small contribution is left. The Brueckner effect slowly decreases above $k = 0.5$ GeV/c, but its contribution remains important within the displayed momentum scale. ($k = 0.75$ GeV/c corresponds to $E \simeq 250$ MeV). Furthermore we compared our results with the Brueckner calculations of Shakin and collaborators [7] (dotted curve). This is not a straightforward comparison however, due to essential differences. They do not use effective Dirac spinors in eq. (1) but free ones, in combination with a different approach to the effective single-particle energies in the intermediate nucleon states. In another calculation with effective Dirac spinors but free single-particle energies above the Fermi sea they find considerable deviations on the nucleon selfenergies. Furthermore rhey use the HEA potential, which originally contains pseudo-scalar pion exchange [14]. It is seen that with respect to Σ_s and Σ_0 our results are larger for the real part (and closer to the empirical values [1], i.e. $\Sigma_s(k = 0.65) = -0.4$ GeV, $-\Sigma_0(k = 0.65) = 0.3$ GeV) but smaller for the imaginary part. We completely disagree on Σ_v, for which our calculations give a much smaller contribution and even a different sign for $\mathrm{Re}(\Sigma_v)$. The value that we obtain for $\Sigma_v(k_F)$ is in agreement with ref. [8].

In order to check the accuracy of our approximation in eq. (4) where we neglected the momentum dependence of the selfenergy Σ and used for the single-particle energies the approximate expression

$$\widetilde{E} = (k^2 + m^{*2})^{1/2} - \Sigma_0(k_F) - m_N \,,$$

we make a comparison with the full single-particle energy which results from relativistic Dirac theory:

$$E = \{k^2[1 + \Sigma_v(k)]^2 + [m_N + \Sigma_s(k)]^2\}^{1/2} - \Sigma_0(k) - m_N \,.$$

Here we use the selfenergy values that are displayed in fig. 1. E and $\widetilde{E}$ are shown in fig. 2. The apparent discrepancy is somewhat misleading since the DB-equation contains only single-particle-energy *differences.* Therefore we may shift the energy spectrum with a constant value. In fig. 2 we shifted E with the constant $\delta E = \widetilde{E}(p = 0.46) - E(p = 0.46)$ (dashed curve), where the value of p is determined by the incoming energy. Furthermore we compare the spectrum with the free energy, $E^0 = (k^2 + m_N^2)^{1/2}$ (dotted line). Since the final results are not very sensitive to the single-particle spectrum [15] we may expect from the rather small discrepancy between the full and the dashed curve an inaccuracy of our results of only a few MeV.

The effect of Pauli blocking and of the full DB contributions at different densities is shown in tables 2, 3.

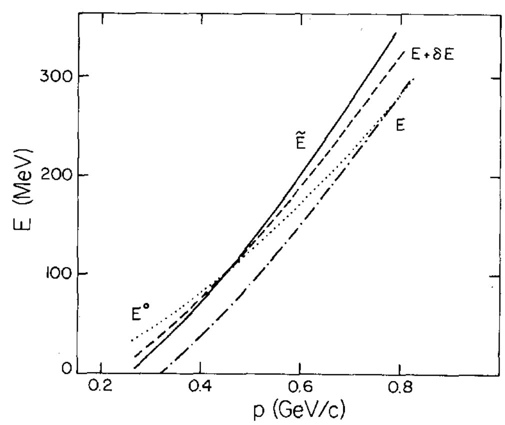

Fig. 2. The single-particle energy as a function of the momentum p. Displayed are the full Dirac result E and the approximate energy $\widetilde{E}$ that enters the Brueckner equation. For a better comparison E has been shifted by a constant δE (dashed line). The free energy E^0 is given by the dotted curve.

We calculated for this the coefficients $a_i(\rho, p)$ and $b_i(\rho, p)$, defined by, respectively,

$$\Sigma_i^{\mathrm{Pauli}}(\rho, p) = [1 - a_i(\rho, p)]\, \Sigma_i^{\mathrm{free}}(\rho, p) , \qquad (9)$$

$$\Sigma_i^{\mathrm{DB}}(\rho, p) = [1 - b_i(\rho, p)]\, \Sigma_i^{\mathrm{free}}(\rho, p) , \qquad (10)$$

where Σ^{free} corresponds to the calculation without any medium effect, Σ^{Pauli} to the calculation that only includes the Pauli blocking and Σ^{DB} to the full DB result. The values that we obtain by eq. (9) can be compared to the results presented by Horowitz [6], i.e. at $\rho = \rho_0$ and $E = 200$ MeV: $\mathrm{Re}(a_s) = -0.01$, $\mathrm{Im}(a_s) = 0.10$, $\mathrm{Re}(a_0) = 0.03$, $\mathrm{Im}(a_0) = 0.20$. For the scalar part of the selfenergy we see a larger effect. The $\rho^{2/3}$-dependence of the coefficients a_i, that he assumed based on phase-space arguments, is roughly reproduced. Also from tables 2, 3 the relative importance of the full DB contribution compared to the Pauli blocking becomes very clear.

The nucleon selfenergy is a non-local quantity since it depends on the momentum k. To make a connection to a local optical potential we calculate the so-called Schrödinger-equivalent optical potential [16], given by

Table 2
The Pauli blocking coefficients $a_i(p, \rho)$

		p(GeV/c)			
		0.46	0.55	0.65	0.75
$\frac{1}{4}\rho_0$	$\mathrm{Re}(\Sigma_s)$	0.04	0.02	0.01	0.01
	$\mathrm{Im}(\Sigma_s)$	0.1	0.1	0.1	0.0
	$\mathrm{Re}(\Sigma_0)$	0.05	0.02	0.00	0.00
	$\mathrm{Im}(\Sigma_0)$	0.1	0.1	0.1	0.1
$\frac{1}{2}\rho_0$	$\mathrm{Re}(\Sigma_s)$	0.08	0.03	0.02	0.01
	$\mathrm{Im}(\Sigma_s)$	0.13	0.11	0.09	0.06
	$\mathrm{Re}(\Sigma_0)$	0.10	0.04	0.03	0.01
	$\mathrm{Im}(\Sigma_0)$	0.17	0.14	0.12	0.10
$\frac{3}{4}\rho_0$	$\mathrm{Re}(\Sigma_s)$	0.09	0.05	0.03	0.02
	$\mathrm{Im}(\Sigma_s)$	0.25	0.17	0.13	0.10
	$\mathrm{Re}(\Sigma_0)$	0.11	0.07	0.04	0.02
	$\mathrm{Im}(\Sigma_0)$	0.28	0.17	0.14	0.13
ρ_0	$\mathrm{Re}(\Sigma_s)$	0.14	0.07	0.04	0.02
	$\mathrm{Im}(\Sigma_s)$	0.31	0.20	0.15	0.12
	$\mathrm{Re}(\Sigma_0)$	0.17	0.08	0.04	0.03
	$\mathrm{Im}(\Sigma_0)$	0.34	0.24	0.19	0.16

Table 3
The Dirac–Brueckner coefficients $b_i(p, \rho)$

		p(GeV/c)			
		0.46	0.55	0.65	0.75
$\frac{1}{4}\rho_0$	$\mathrm{Re}(\Sigma_s)$	0.18	0.16	0.14	0.13
	$\mathrm{Im}(\Sigma_s)$	0.39	0.33	0.26	0.24
	$\mathrm{Re}(\Sigma_0)$	0.22	0.19	0.17	0.15
	$\mathrm{Im}(\Sigma_0)$	0.40	0.33	0.29	0.27
$\frac{1}{2}\rho_0$	$\mathrm{Re}(\Sigma_s)$	0.29	0.25	0.24	0.22
	$\mathrm{Im}(\Sigma_s)$	0.61	0.51	0.43	0.36
	$\mathrm{Re}(\Sigma_0)$	0.33	0.29	0.27	0.25
	$\mathrm{Im}(\Sigma_0)$	0.61	0.53	0.45	0.39
$\frac{3}{4}\rho_0$	$\mathrm{Re}(\Sigma_s)$	0.34	0.33	0.30	0.28
	$\mathrm{Im}(\Sigma_s)$	0.75	0.64	0.52	0.44
	$\mathrm{Re}(\Sigma_0)$	0.38	0.36	0.34	0.31
	$\mathrm{Im}(\Sigma_0)$	0.74	0.63	0.54	0.47
ρ_0	$\mathrm{Re}(\Sigma_s)$	0.40	0.37	0.35	0.34
	$\mathrm{Im}(\Sigma_s)$	0.82	0.70	0.59	0.48
	$\mathrm{Re}(\Sigma_0)$	0.43	0.40	0.38	0.36
	$\mathrm{Im}(\Sigma_0)$	0.81	0.70	0.60	0.51

$$U_c(E) = \Sigma_s(k) - (E/m_N + 1)\Sigma_0(k) + [\Sigma_s(k)^2 - \Sigma_0^2(k)]/2m_N , \qquad (11)$$

as a function of the single-particle energy E.

Following the same calculational procedure as before, we present our results in fig. 3, where they are compared to empirical Woods–Saxon well depths. This comparison has of course only a limited validity, especially at higher energies [7]. It is seen that the full Brueckner results fall nicely in line with the empirical values. Note that in fig. 3 we did not rescale $\mathrm{Im}(U_c)$ with an effective-mass factor $(\widetilde{m}/m_N)$. Compared to ref. [7] the results differ at higher energies, where in our case $\mathrm{Re}(U_c)$ is somewhat larger, where $\mathrm{Im}(U_c)$ is smaller in magnitude. We might note here that our $\mathrm{Re}(U_c)$ is rather similar to the optical potentials obtained in non-relativistic Brueckner calculations [15, 18], which however overpredict the imaginary part of the optical potential. The similarity holds also, if we look at the density dependence of U_c, which is displayed in fig. 4. As in the non-relativistic case, at low energies the potential at normal density ρ_0 is more attractive than at $\frac{1}{2}\rho_0$, while above $E \sim 150$ MeV it becomes the inverse. A variational calculation

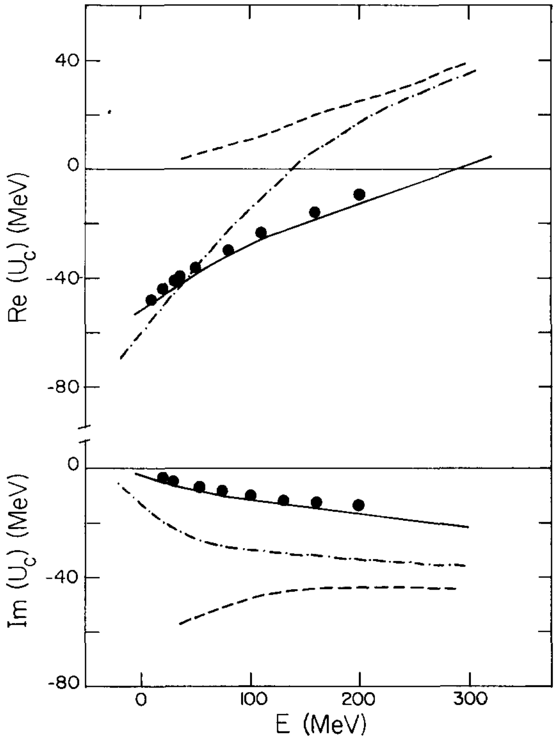

Fig. 3. The Schrödinger-equivalent optical potential as a function of the single-particle energy E. The curves have the same representation as in fig. 1. The data points show empirical Wood–Saxon depths. The variation in the empirical values is comparable to the size of the points in this figure (see e.g. ref. [17]).

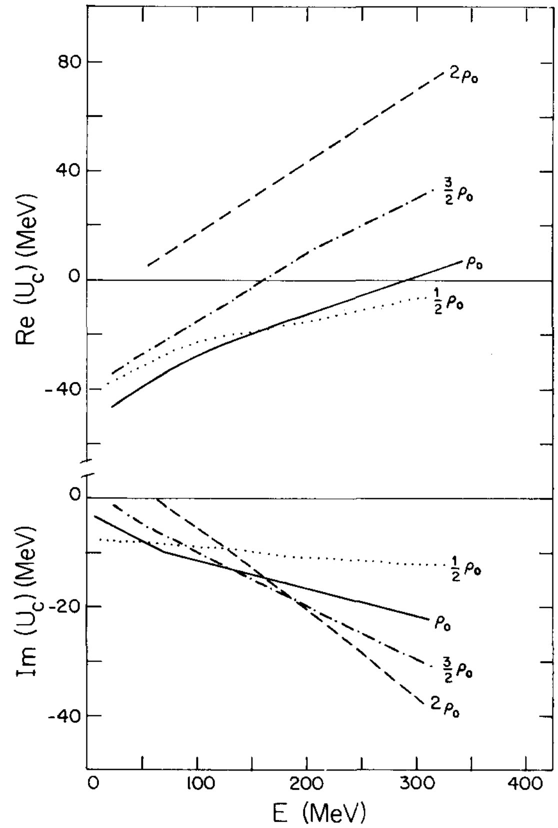

Fig. 4. The Schrödinger-equivalent optical potential at different nuclear-matter densities within the Dirac–Brueckner approach.

by Friedman and Pandharipande [17] shows the same behaviour, but has a somewhat lower crossing point. The results at higher densities roughly agree with the phenomenological mean-field calculations of Boguta [19].

In conclusion we studied the single-nucleon interaction in nuclear matter at particle energies up to 250 MeV, within the relativistic Dirac–Brueckner approach. We separated the different medium effects and showed that at the lower end Pauli blocking dominates, but at the higher end of our energy scale the Brueckner effect is much more important. This led at 200 MeV in a Lorentz scalar attraction of −320 MeV and a Lorentz vector repulsion of 230 MeV. We deduced a Schrödinger-equivalent optical potential which turned out to be rather similar to non-relativistic Brueckner results. Also, the density dependence of this potential falls in line with earlier calculations.

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